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ABSTRACT

Thirty-six prospective elementary education teachers were randomly assigned to three instructional moves: a characterization-exemplification with declarative statements (CED), a characterization-exemplification with interrogative statements (CEI), and an exemplification (E). The purposes of the study were to: (1) examine the efforts of the three sequences of moves whenever the number of exemplification moves for the CE sequences was increased to that used by the E sequence, and (2) to determine the differential effects of activity and passivity learning relative to the three sequences of moves. Programmed instructional materials were used to develop a partitioning and a measuring-pyramid concept for the three-day experimental period. The only significant differences detected were those between the high and low achievement level groups. The CE sequences were found to be as effective as the E sequences, and passivity learning was found to be as effective as activity learning at the concrete and symbolic stage. (Author/MN)

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A Comparison of Three Sequences of Moves for Teaching Conjunctive and Relational Mathematical Concepts to Students in Elementary Education

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Abstract

Thirty-six prospective elementary education teachers were randomly assigned to three instructional moves: a characterization-exemplification with declarative statements (CE_D), a characterization-exemplification with irterrogative statements (CE_I), and an exemplification (E). The purposes of the study were to (1) examine the efforts of the three sequences of moves whenever the number of exemplification moves for the CE sequences was increased to that used by the E sequence and (2) to determine the differential effects of activity and passivity learning relative to the three sequences of moves. Programmed instructional materials were used to develop a partitioning and a measuring-pyramid concept for the three-day experimental period. The only significant differences detected were those between the high and low achievement level groups. The CE sequences were found to be as effective as the E sequences, and passivity learning was found to be as effective as activity learning at the concrete and symbolic stage.

A Comparison of Three Sequences of Moves for Teaching Conjunctive and Relational Mathematical Concepts to Students in Elementary Education

Recent studies (Dossey and Henderson, 1974; Gaston and Kolb, 1973; Rector and Henderson, 1970; Catanzano and Godwin, 1976, 1977) have applied the Henderson (1967) model to ascertain the relative effects of different sequences of moves on certain mathematical concepts classified as conjunctive, disjunctive, or relational by Bruner's (1956) classification scheme. Some results from these investigations suggest that (1) Exemplification-Characterization-Exemplification (ECE) sequence is not adequate for developing mathematical concepts; (2) the use of exemplification sequences of moves appears sufficient for the learning of certain mathematical concepts; (3) the E sequence appears to be more effective for the learning of conjunctive concepts than other sequences whenever the number of exemplification moves is sufficient for concept formation; (4) the Characterization-Exemplification-Characterization (CEC) (or EC) sequence appears to be the most effective for learning disjunctive concepts; (5) students (college level) are capable of primitive generalization, thus examples using limited sets of numbers can be used effectively for teaching conjunctive and relational mathematical concepts.

Gaston and Kolb in their 1973 study used thirty-eight exemplification moves for the E sequence and only three for the CE sequence. Catanzano and Godwin (1977) reduced the number of exemplification moves of the E sequence to thirteen and increased the number for the CE sequence to four and found the E sequence to be superior, a result similar to that found by Gaston and Kolb.

Special commendation is given to Ms. Debbie Welker for her assistance with the preparation and execution of the study.

Whether or not these findings will remain supportive when the number of exemplification moves of the CE sequence is increased is subject to investigation. Such investigation should help reveal the nature of exemplification moves; that is, is it the number of exemplification moves, or is it the abstracting process whereby the student is required to identify the concept from the exemplars of the referent set--class formation--that has produced the effect?

Activity learning is a global concept often defined to include all learning settings in which a concept is developed by the active participation of the learner (Kieren, 1969). Active participation of the learner means that the learner is directed by the teacher via a sequence of questions, directions, problems, or manipulative materials to act on (identify) a mathematical concept from exemplars. If, on the other hand, the teacher identifies the concept for the student by explanation, telling, or showing while the student passively listens, then passivity learning may take place. Whether one approach is superior to the other for the learning of concepts is open to question (Kieren, 1971). An application of the Henderson model to these variables should help provide a way to objectively identify the variables associated with mathematical instructional theory.

Problem

Three sequences of moves were defined: a characterization-exemplification (CE_D) move having only declarative statements (passivity learning), a characterization-exemplification (CE_I) move having declarative and interrogative statements (passivity learning with student responses), and an

exemplification (E) move having declarative and interrogative statements (activity learning). The purposes of this study were (1) to examine the effects of the three sequences of moves whenever the number of exemplification moves for the CE sequence was increased to that used by the E sequence and (2) to determine what differential effect activity learning (at the manipulative and nonmanipulative stages) and passivity learning have on the immediate acquisition and short-term retention of conjunctive and relational concepts.

An attempt to examine the relative merits of these sequences on the immediate acquisition and short-term retention of two mathematical concepts--partitioning, pyramid--required investigating the following set of hypotheses for each criterion measure:

H_1: The mean scores of the three treatment groups are equal; that is, $\mu_{CE_D} = \mu_{CE_I} = \mu_E$.

H_2: The interaction effect between treatments and achievement levels is zero; that is, $I_{TA} = 0$.

H_3: The mean scores for the two ability levels for each treatment are equal; that is, $\mu_{Low} = \mu_{High}$.

As areas of minor interest, answers to the following questions were desired:

1. What is the length of instructional time for each sequence of moves?
2. Which concept was chosen by the students in each group to be the easiest to learn?
3. Were the students in each group capable of recognizing that the same type of sequence of moves was applied to them during the experimental period?

4. Did the groups enjoy learning their topics?

5. Did the groups like having each lesson written, read, and studied individually during the class meeting?

Methodology

Subjects. Thirty-six students enrolled in a course, Mathematics for the Elementary Teacher, at Berry College were blocked into four groups according to their outcomes on two measures: (1) achievement on a pretest measuring set concepts and (2) M-SAT College Entrance Examination Board scores. The achievement pretest covering prerequisite knowledge for the partitioning concept under study was the primary factor used for the blocking. Students in each of the four groups were randomly assigned to the three treatments. In order to maintain a reasonable number of subjects in each cell for the analysis, the two upper and the two lower groups were collapsed. Thus, following random assignment of groups to treatments, three treatment groups were established, each containing twelve subjects--seven in the upper group and five in the lower.

All students were classified below the senior level with the following distribution of M-SAT scores: 22% between 500 and 550, inclusive; 17% between 450 and 499, inclusive; 22% between 400 and 449, inclusive; 17% between 350 and 399, inclusive; 21% below 350; 1% with no records. With two exceptions, the students classified in the low achievement level group had M-SAT scores below 401.

Concepts. Two concepts were selected for this study according to the following criteria: (1) one concept should be that used in the Gaston and Kolb and Catanzano and Godwin studies; (2) student familiarity with the concepts should be slight or nil; (3) one concept should be part of the sequel for the course; and (4) the one satisfying criterion (3) should also be capable of development through manipulative devices. The partitioning concept satisfied criteria (1) and (2). The pyramid concept satisfied criteria (2), (3), and (4). The definitions used in this study were: (a) A partition of a given set A is a set of nonempty subsets of A such that the union of these subsets is the given set A, and the intersection of each pair of the subsets is the empty set; (b) In order to find the measure of the volume of a (right) rectangular pyramid, calculate the measure of the area of the base, multiply the result by the height of the pyramid, then multiply by one-third; thus, the open sentence for finding the measure of the volume of a rectangular pyramid is $1/3Bh$.

According to Bruner's scheme for classifying concepts, the partitioning concept was classified as conjunctive; the pyramid concept as relational.

Treatments. By applying the Henderson model, three sequences of moves were developed for each concept, producing six variations. For the partitioning concept the following sequences of moves were used: characterization-exemplification with declarative statements (CE_D)--those not requiring a student response, characterization-exemplification with interrogative statements (CE_I)--those requiring a student response, and exemplification (E). The same sequences were applied to the pyramid concept but with fewer moves. Another variation between the moves for the parti-

tioning concept and those of the pyramid concept pertained to the E sequence. The E sequence of the former used examples in the form of written examples from which the student derived the attribute; the E sequence of the latter used examples in the form of manipulative containers, a pyramid and a prism of comparable dimensions, from which the student derived the relationship. Although the sequences of moves can be classified into three distinct types, they varied by the number of characterization and exemplification moves applied as defined below.

1. The CE_D sequence contained only declarative statements. Ten moves were used for the partitioning concept: a characterization, followed by an example with justification, followed by a nonexample with justification, followed by an example with justification, a nonexample with justification, an example with justification, a nonexample with justification, an example with justification, two nonexamples with justification. Four moves were used for the pyramid concept; two characterizations (the first being a prerequisite review move), two examples with justification.
2. Except for a declarative characterization move, the CE_I sequence contained all interrogative statements. Ten moves paralleling those of the CE_D sequence were used for the partitioning concept; four paralleling those of the CE_D were used for the pyramid concept.
3. The E sequence contained both declarative and interrogative statements. The declarative statements were presented as examples from which the student had to induce the concept.

Fifteen moves, the initial eight declarative and remaining seven interrogative, were used for the partitioning concept: four examples, followed by four nonexamples, followed by an example, a nonexample, an example, a nonexample, an example with justification, two nonexamples with justification. A maximum of nine moves was possible for the pyramid concept. The initial move was a characteristic prerequisite review move; the following three moves were questions requiring an examination of the manipulative materials for the identification of the given conditions of the concept; (the following two example moves were applied only if the relationship was not induced from the previous moves by applying the manipulative materials); a charactermization interrogative move was used to insure knowledge of the derived relationship; the next two were interrogative example moves.

Although the number of moves for the CE_D and CE_I sequences differed by five from that of the E sequence for the partitioning concept, the last seven moves of each contained the same examples, and the first two examples of the CE_D and CE_I were taken directly from the initial eight declarative examples of the E sequence. It seemed superfluous to repeat examples using the same exemplification moves, the only difference being the declarative-interrogative dichotomy and the class identification-example identification dichotomy. Except for the number of moves the CE_D and E sequences used in this study paralleled those used in the Catanzano and Godwin and Gaston and Kolb studies.

In order to induce the pyramid concept from examples, the E sequence was applied to manipulative objects. The CE_D and CE_I sequences used definitions, pictures, and examples.

Procedure. On the second day preceding the last day of the fall quarter, 1974, the experimental class was partitioned into three randomly selected groups. Portable partitions were used to separate the room into three areas so that each group was essentially self-contained. Instructional materials prepared according to the defined treatments were issued to each of the three groups by two instructors, the experimentor and an assistant. The instructors served as proctors and assistants in those cases where help was needed. Since the instructional materials were prepared in a programmed format, very few instructions were necessary except in those cases where interrogative statements appeared. In such cases instructors accepted the answer as being right or indicated to the student that he should read a portion of the material again. The reasons for not supplying answers to the interrogative statements was (1) that the instructional setting should assimilate the real setting as much as possible, (2) that the treatments may be confounded with one another since distinctions were made between interrogative and declarative statements, and (3) that the manipulative materials used for the pyramid concept were programmed in a looping style. The number of instructor-student contacts were approximately equally distributed among the three groups. As soon as a student completed the instructional material, he was given the immediate acquisition posttest. Two days were used to administer the treatments and the immediate acquisition posttests--the partitioning concept was given on the first day, the pyramid concept on the second day. On the

third day a questionnaire and a short-term retention posttest were administered in that order.

Five individual learning booths were prepared for the E group of the pyramid concept. Each booth contained a pyramid, a prism of equal height and congruent base, a funnel, and a container of sand. Students were instructed to enter the booths one at a time to complete the assigned tasks. If they were unable to apply the materials in a way that produced the desired results--namely that $A = 1/3\ Bh$--then a special sheet instructing them to pour the sand into the pyramid and then into the prism to make the desired comparisons was issued. Since only five booths were available for twelve students, this procedure proved to be ineffective in terms of instructional time and task assimilation. Surprisingly, only 20% of the students were able to perform the task without being administered the special sheet.

No assignments or feedback from posttests were made during the experimental period. The students were told that they would be tested each day following the instructional period, but were not told that they would receive a short-term retention posttest on the last day of the experimental period.

Instrumentation. The achievement pretest was designed to measure the student's understanding of set notation, set relations, and set operations at the primitive generalization and vertical transfer levels. Following approximately two weeks of instruction on those topics during the beginning of the fall quarter, the achievement pretest was administered and its results were used to block the experimental group into upper and lower achievement levels. This pretest was the same as that used in the Catanzano and Godwin study (1977).

Immediate acquisition criterion tests were given each day following the treatments. The immediate acquisition test for the partitioning concept consisted of three subtests. Subtest I was composed of six problems taken directly from those given during instruction, the lowest level of assessment. Some of the problems were reversed; that is, if presented "given A, find B", then the new question would be "find A, given B". The purpose of subtest II was to determine the degree of primitive generalization (Dienes, 1961). Subtest II consisted of twenty-one problems with replacement sets different from those used during instruction. Subtest III consisted of thirteen upper level problems requiring students to transfer vertically (Gaston and Kolb, 1973) or involving the process of abstraction (Dienes, 1961). Only two immediate acquisition subtests were used for the pyramid concept: subtest II containing six problems and subtest III containing ten problems.

The short-term retention test was administered on the third day, following the instructional period and the administration of the questionnaire. This test was composed of twenty items, fifteen for the partitioning concept and five for the pyramid concept. The test items were classified at the primitive generalization and vertical transfer level.

Special emphasis was given to the content validity of all criterion tests as outlined above. Test-retest external reliability of stability was accepted by inference since most of the items were the same as those used in the Catanzano and Godwin studies (1967,1977). All pretests and posttests used a completion type format.

Analysis and Findings

Inferential results. A 2 x 3 factorial design with proportional but unequal cell sizes (Winer, 1971, p. 419) was used to ascertain the effects of the three sequences of moves at low and high achievement levels for each of two concepts. The assumption of homogeniety of variance verified by the Hartley F-max test was accepted for each criterion measure. Means for each cell and number of test items are listed in Table 1.

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Insert Table 1 about here
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By applying the analysis of variance to the scores on the immediate acquisition subtests and short-term retention test of the partitioning concept, the only significant ratios detected were those between achievement levels (See Table 2). Further analysis revealed that significant differences (observed t > critical $t_{.05}$ = 2.04 with df = 30) existed between the high and low achievement levels for the CE_D sequence and the CE_I sequence of subtest I, for the CE_D sequence and E sequence of subtest II, for the CE_D sequence and CE_I sequence of subtest III, and for the CE_D sequence and CE_I sequence of the short-term retention test. No other significant differences were detected.

Similar findings were detected for the pyramid concept shown in Table 3. However, an analysis of these differences revealed that the E sequence was the only treatment for which differences existed between the high and low achievement levels.

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Descriptive results. In order to provide information about the instructional setting and to provide support for the inferential findings, a descriptive analysis of a student questionnaire and the instructional phase of the study was made. The instructional time was defined to be the time that it took all students to complete the reading and responsive portion of the instructional material. The instructional time for the partitioning concept was: 13 minutes for the CE_D sequence, 5 minutes for the CE_I sequence, and 30 minutes for the E sequence. The instructional time for the pyramid concept was: 4 minutes for the CE_D sequence, 5 minutes for the CE_I sequence, and 30 minutes for the E sequence.

The students in the CE_D group selected by a ratio of 7 to 5 the partitioning concept to be the easiest to learn; similarly, those in the CE_I and E groups selected by a ratio of 8 to 4 the partitioning concept. Students in the CE_D and CE_I groups indicated by a ratio of 10 to 2 that they received the same type of sequence of moves during the experimental period; the E group indicated that the partitioning concept was presented by examples, while the pyramid concept was presented through experimentation. This difference was detected 9 out of 12 times. In each case evidence was given that supported the contention that information was presented in such a way as to make the learning pupil-initiated. Eight out of twelve students in the CE_D group indicated that they enjoyed learning their topics, four out of twelve in the CE_I group and ten out of twelve in the E group indicated a positive reaction to this. Six out of twelve students in the CE_D group indicated that they liked having the lesson written, read, and studied individually; five out of twelve from the CE_I and seven out of twelve from the E group indicated a positive reaction to this item.

All students indicated that they had not prepared for any of the class meetings by studying for the mathematical topics presented, and approximately 15 percent indicated that they had discussed this topic with other members of the class. Only one student indicated that he had studied the topics presented in this study in other mathematics courses.

In connection with the preparation of the instructional material, the E sequence of the pyramid concept was found to be the most difficult to prepare in comparison with CE_D and CE_I sequences because of the necessity of having manipulative materials. In terms of the execution of the instructional material, the E and CE_I sequences required more instructor time than the CE_D sequence. Furthermore, the E sequence of the pyramid concept required more instructor time than all other sequences.

Conclusion and Discussion

On the basis of the inferential findings reported in the previous section some general conclusions related directly to the hypotheses and questions of this study can be drawn for the specified sample used.

1. The findings supported the hypothesis for each of the two concepts that the three sequences of moves were equally effective when achievement was measured by immediate acquisition subtests, their composite, and short-term retention tests.
2. No significant interaction effects were detected.
3. Significant differences were found between the high achievement level group and the low achievement level group for the partitioning concept on all subtests, composites, and short-term retention tests except for the E sequence of subtest I, the CE_I sequence of subtest II, and the E

sequence of subtest III and short-term retention test. For the pyramid concept, the only significant differences found were those between the high and low achievement level groups of the E sequences on the criterion tests.

In connection with the descriptive findings,

4. The instructional time for each sequence of moves was found to be approximately the same for the partitioning concept but was found to be greater for the E sequence than for either the CE_D or CE_I sequences for the pyramid concept.
5. The partitioning concept was selected more often than the pyramid concept as the easiest to learn.
6. Most of the students recognized that they were taught by the same type of sequence of moves.
7. More than two-thirds of the students in the CE_D and E groups indicated that they enjoyed learning the assigned topics. Less than one-half indicated this attitude from the CE_I group.
8. Only students from the E group (more than half) favored the programmed instructional method.

The evidence seems to suggest rather strongly that the superiority of the E sequence on the immediate acquisition of conjunctive concepts reported by Gaston and Kolb (1973) and Catanzano and Godwin (1977) was attributable to the number of exemplification moves employed (and not to the process of concept formation). This conclusion is also supported by that found by Catanzano and Godwin (1977) when the number of exemplification moves used for the relational concepts of that study are compared. Since Gaston and Kolb used 38 exemplification moves for the E sequence and Catanzano and Godwin

used only 13 to achieve the same results, there appears to be an optimum number of moves necessary for the promoting the immediate acquisition of concepts. Although the optimum number will perhaps vary depending on the nature of the concept and the nature of the student, the development of an E sequence designed to identify a concept when given a sequence of exemplars of that concept should help identify the nature (and minimum number) of the exemplification moves required. It may be that whenever an optimum number of exemplification moves is used in a CE or other type of sequence containing a characterization move, the effect of the characterization move is lost. This possibility needs to be examined.

The differences found between the high and low achievers do not agree with the results found by Cantanzano and Godwin (1977) on this factor. Essentially no significant differences on immediate acquisition subtests were detected by Catanzano and Godwin (except for the isomorphic concept). Whereas, in this study, the differences between the high and low ability groups of the E sequence of the partitioning and pyramid concepts were the only ones found to be insignificant and significant, respectively. This reversal trend may have occurred because of the method of teaching employed. For instance, (1) the instructional procedure used for the E sequence of the pyramid concept required a higher degree of open search for the answer and (2) the instructional setting had an insufficient number of learning booths available, thereby increasing the instructional time to 25 minutes. Consequently, several students (the low achievers primarily) had to remain beyond the normal class period in order to complete the immediate acquisition posttest. Thus, this discrepancy in procedure may have influenced the results pertaining to the high and low achiever in the E treatment group.

In connection with the results pertaining only to the partitioning concept of the two studies, it appears that the E treatment promotes the immediate acquisition of conjunctive concepts for the low achiever as well as it does for the high achiever. Due to the importance of this result for teaching, this problem needs further study.

Studies assessing the effects on the immediate acquisition and short-term retention of concepts may differ drom those related to the long-term retention of concepts. Thus, the long-term retention and immediate acquisition variables need to be incorported in experimental studies related to teaching.

Implications for Instructional Practice

In connection with research findings related to the Henderson model, several trends in terms of teaching mathematics at the college level (and perhaps at lower levels) appear relevant.

1. Attitude of students from the E sequence group toward learning and the type of instruction appears to be more favorable than those from the CE_D and CE_I sequences.
2. The number of exemplification moves (examples and nonexamples with and without justification) seems to be the primary factor that affects the immediate acquisition of conjunctive (and possibly relational and disjunctive) concepts. Although the optimum number of exemplification moves varies from concept to concept, the number needed for a given concept appears to be at least equivalent to the number required by the student to identify the concept from exemplars of the concept, that is, the process of class formation. Not only does it appear that the number of exemplification moves is critical but also that the moves

be continuous, that is, with no characterization moves interrupting the chain of exemplification moves. Whenever the optimum number of continuous examples are used, then the type of sequence of moves appears irrelevant; this is, E, CE, CEC, and EC sequences are equally effective. Similarly, under optimum conditions of exemplification, passivity learning is as effective as activity learning, whenever a programmed format designed for individualized instruction is applied. It may be that activity learning is more effective than passivity learning in a typical instructional setting that does not incorporate programmed units. Thus, under these conditions activity learning may motivate each student by gaining his attention whereas passivity learning may or may not obtain a similar amount of concentration from each student.

3. The E sequence appears to be the most effective for the immediate acquisition of conjunctive concepts whenever the number of exemplification moves is sufficient for class formation and exceeds by at least threefold that used by other sequences.

4. Since the optimum number of exemplification moves may vary from student to student, it appears that interrogative examples must be presented following a sequence of declarative examples in order to assess the degree of concept acquisition from which a decision to give more examples (declarative or interrogative) is made.

5. Students (college level) appear capable of primitive generalization; thus, using examples (rather than characterization moves) with a relatively small universal set to introduce a concept is sufficient to promote the acquisition of the concept.

6. A surprising observation derived from this experiment was the inability of the E sequence group to perform a manipulative task without being given additional clues. This lack of competence seems particularly significant for a group of prospective elementary teachers whose training should include a sensitization to concrete-manipulative devices and an awareness of presenting concepts at the concrete or manipulative level. For instance, presented with a problem, how can it be solved, first at the symbolic level and second at the manipulative level?

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Table 1

Means and Number of Test Items for Each Subtest, Composite, and Retention Test for the High and Low Achievement Groups of Each Concept

Treatments	Immediate acquistion test[a]								Short-term retention test[a]	
	Subtest I		Subtest II		Subtest III		Composite			
	high	low	high	low	high	low	high	low	high	low
Partitioning										
	(6)		(21)		(13)		(40)		(15)	
CE_D	5.86	4.80	19.43	16.60	7.43	2.80	32.71	24.20	10.57	6.80
CE_I	6.00	5.20	19.00	17.00	8.29	4.60	33.29	26.80	11.43	7.00
E	5.43	4.80	19.57	17.00	7.86	5.60	32.86	27.40	10.43	7.80
Pyramid										
			(6)		(10)		(16)		(5)	
CE_D			5.00	4.00	6.71	6.00	11.71	10.00	4.00	3.40
CE_I			4.57	4.20	6.57	6.00	11.14	10.20	3.71	2.60
E			5.00	3.40	7.57	4.80	12.57	8.20	4.57	2.80

[a]Numbers in parentheses represent total number of test items.

Table 2

Analysis of Variance of the Scores on the Immediate Acquisition Subtests and Composite and the Short-term Retention Test of the Partitioning Concept

Source of variation	df	MS	F
	Immediate acquisition subtest I		
Treatments (T)	2	.75	1.19
Achievement level (A)	1	6.01	9.54*
TA	2	.14	.22
Within cell	30	.63	
	Immediate acquisition subtest II		
Treatments (T)	2	.36	.08
Achievement level (A)	1	48.24	11.24*
TA	2	.53	.12
Within cell	30	4.29	
	Immediate acquisition subtest III		
Treatments (T)	2	7.20	.86
Achievement level (A)	1	108.66	12.98*
TA	2	4.16	.50
Within cell	30	8.37	
	Immediate acquisition composite test		
Treatments (T)	2	8.03	.40
Achievement level (A)	1	406.87	20.20*
TA	2	7.06	.35
Within cell	30	20.15	

Table 2 - Continued

Source of variation	df	MS	F
	Short-term retention test		
Treatments (T)	2	1.03	.14
Achievement level (A)	1	114.00	15.36*
TA	2	2.42	.33
Within cell	30	7.42	

*$p < .05$

Table 3

Analysis of Variance of the Scores on the Immediate Acquisition Subtests and Composite and the Short-term Retention Test of the Pyramid Concept

Source of variation	df	MS	F
Immediate acquisition subtest II			
Treatment (T)	2	.20	.17
Achievement level (A)	1	8.59	7.67*
TA	2	1.10	.98
Within cell	30	1.12	
Immediate acquisition subtest III			
Treatment (T)	2	.03	.01
Achievement level (A)	1	16.01	4.30*
TA	2	4.42	1.19
Within cell	30	3.72	
Immediate acquisition composite test			
Treatment (T)	2	.25	.04
Achievelent level (A)	1	48.03	7.29*
TA	2	9.44	1.43
Within cell	30	6.59	
Short-term retention test			
Treatment (T)	2	1.20	.99
Achievement level (A)	1	11.82	9.77*
TA	2	1.01	.83
Within cell	30	1.21	

*$p < .05$